

# Relativistic Split-Cavity Oscillator

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## Abstract

Using the method of small signal analysis, we study the application potential of relativistic electron beams in split-cavity oscillators (SCO's). A beam-energy change in the SCO as a function of the initial energy of a relativistic beam is considered. It is shown that the small-signal analysis method enables adequate evaluation of SCO parameters needed for effective modulation of a relativistic beam in a split cavity and for HPM generation using SCO's. The range of energies is found for which the effect of self-modulation of the beam density in SCO structures is most pronounced. It is also shown that for beam currents at which the space charge has little effect on the motion of electrons in a beam, the beam in a split-cavity oscillator is effectively self-modulated at beam energies less than  $\approx 300 \div 400$  keV. The self-modulation drops sharply in the range of energies from 250 to 400 keV, but as the beam current is increased, the effective beam self-modulation becomes appreciable in this range too, as well as even in a higher energy range.

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## 1 Introduction

The study of radiation generation mechanisms via the transit-time effect is one of the lines of research into the process of high-power microwave generation. Transit-time effect oscillators (TTO) (monotron [1,2,3], split-cavity oscillator (SCO) [4], super-reltron [5], double-foil SCO [6,7]) enabled the HPM generation using high-current electron beams without the need for an applied magnetic field.

In a fundamental work [4], Marder and colleagues not only gave a detailed computer simulation of SCO behavior, but also developed in the case of small signal analysis a method allowing the evaluation of SCO parameters required for effective modulation of nonrelativistic electron beam in SCO's (for effective HPM generation through the use of SCO extractors).

In view of the extensive research into the use of high-current relativistic electron beams for HPM generation, in this paper we shall generalize the small signal analysis method suggested in [4] to the case of relativistic beams passing through a split cavity.

It is shown that the small signal analysis method enables adequate evaluation of SCO parameters needed for effective modulation of a relativistic beam in a split cavity and for HPM generation using SCO's. The range of energies is found for which the effect of self-modulation of the beam density in SCO structures is most pronounced.

## 2 Small Signal Analysis of the SCO

The split-cavity oscillator (SCO) [4] consists of a high-Q pillbox cavity with conducting screen walls through which an electron beam can pass. The cavity is symmetrically partitioned by a screen which leaves a gap between it and the outer cavity wall (see Figure 1).

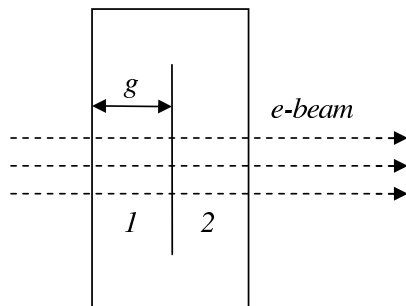


Figure 1. SCO geometry

Following Marder and co-authors [4], to describe the behavior of the SCO in the case of relativistic electron beams, we shall first consider the transit-time oscillator (see Figure 2) excited by a relativistic beam.

The TTO pillbox is like an SCO, but without partition (see Figure 2). According to [4], the pillbox cavity of the TTO can be approximated by a one-dimensional gap of width  $g$  across which a uniform RF electric field of the form

$$\vec{E} = \vec{E}_0 \sin(\omega t + \vartheta) \quad (1)$$

is imposed.

It is assumed that the electric field  $\vec{E}$  is directed along the  $z$ -axis and the electron entering the pillbox cavity moves parallel to the  $z$ -axis;  $\omega$  is the field frequency and  $\vartheta$  is the initial phase of the field (the phase of the field at the

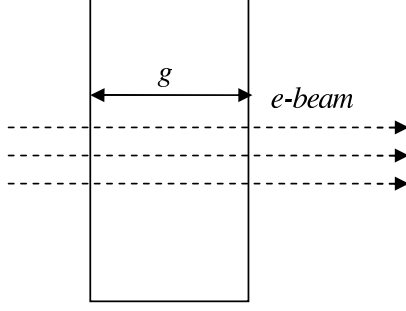


Figure 2. TTO geometry

moment when the particle enters the pillbox cavity). The magnetic field  $\vec{H}$  exists in the pillbox cavity together with the electric field. However, in the case of small signals analysis, when the Larmor radius of particle motion in a magnetic field is much greater than the width  $g$ , the magnetic field only causes the displacement of the electron in the direction orthogonal to the  $z$ -axis, but has no effect on the longitudinal motion of the particle along the  $z$ -axis. In what follows we shall neglect the influence of the magnetic field on particle motion. As a result, the equation of motion along the  $z$ -axis for the electron entering into the pillbox cavity can be written in the form:

$$\frac{dp}{dt} = -e E(t) = -e E_0 \sin(\omega t + \vartheta) \quad (2)$$

where  $p$  is the momentum of the electron and  $e$  is the value of the electron charge;  $p = m\gamma v$ , where  $m$  is the mass of the electron and  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$  is the Lorentz factor, with  $v$  and  $c$  being the electron velocity and the speed of light, respectively.

It follows from (2) that

$$p(t, v) = p_0 + \frac{eE_0}{\omega} [\cos(\omega t + \vartheta) - \cos \vartheta], \quad (3)$$

that is,

$$\begin{aligned}\gamma v &= \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \gamma_0 v_0 \left\{ 1 + \frac{eE_0}{m\gamma_0 v_0 \omega} [\cos(\omega t + \vartheta) - \cos \vartheta] \right\} = F(t, \vartheta),\end{aligned}\tag{4}$$

where  $\gamma_0$ ,  $v_0$  are the particle's Lorentz factor and velocity at the moment of entering the pillbox cavity, respectively.

The solution of (4) has the form

$$v = \frac{dz}{dt} = \pm \frac{F}{\sqrt{1 + \frac{1}{c^2} F^2}}.\tag{5}$$

We shall further be interested in the motion in the positive direction of the  $z$ -axis, so in (5) we shall only consider the solutions with the plus sign. Equation (5) yields the following expression for the trajectory of relativistic-particle motion in a pillbox cavity

$$z(t) = \int_0^t F(t', \vartheta) \frac{1}{\sqrt{1 + \frac{1}{c^2} F(t', \vartheta)}} dt' \tag{6}$$

Let  $T$  denote the time when the electron crosses the right border of the pillbox cavity (Figure 2) ( $T$  - is the moment when the electron escapes from the cavity). According to (6), we have

$$\int_0^T F(t', \vartheta) \frac{1}{\sqrt{1 + \frac{1}{c^2} F(t', \vartheta)}} dt' = g \tag{7}$$

Using Eqs. (6), (7), we can find the time needed for the particle to pass the distance  $g$  in the presence of the field  $E$ .

The function  $F$  depends on the parameter

$$\varepsilon = \frac{eE_0}{m\gamma_0 v_0 \omega}.$$

In the case of small signal analysis, the parameter  $\varepsilon \ll 1$ . Let us expand the integrand in (7) into a power series in the small parameter  $\varepsilon$ . The left-hand side of (7) can be integrated within an accuracy up to the terms proportional to the parameter  $\varepsilon$  raised to the first power, which gives the following expression for the time  $T$ :

$$T = \frac{g}{v_0} - \tilde{\varepsilon} \frac{[\sin(\omega T_0 + \vartheta) - \sin \vartheta - \omega T_0 \cos \vartheta]}{\omega}, \quad (8)$$

where we introduced the following notation:

$$\tilde{\varepsilon} = \frac{1}{\gamma^2} \varepsilon, \quad \varepsilon = \frac{e E_0}{m \gamma_0 v_0 \omega}, \quad T_0 = \frac{g}{v_0}.$$

In a nonrelativistic case, when  $\frac{v_0}{c} \ll 1$ , expression (8) goes over to expression (4) in [4].

With (8) we can find the momentum and then the energy  $W$  of the particle at the moment when it crosses the right border of the pillbox cavity (Figure 2), as well as the difference  $W - W_0$  between the final energy  $W$  of the particle when it leaves the pillbox cavity and its initial energy  $W_0$  when it enters the cavity.

We shall consider the difference  $W^2 - W_0^2$  because the relation  $W^2 = p^2 c^2 + m^2 c^4$  between  $W^2$  and the squared momentum  $p^2$  of the particle is more convenient to use in our further analysis.

For this difference averaged over the initial phase  $\vartheta$  of particle entrance into the cavity, we can obtain the following expression:

$$\frac{W^2 - W_0^2}{p_0^2 c^2} = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{p^2(T, \vartheta)}{p_0^2} - 1 \right) d\vartheta. \quad (9)$$

After some cumbersome calculations, we can recast (9) in the form:

$$\frac{E^2 - E_0^2}{p_0^2 c^2} = (\varepsilon \tilde{\varepsilon} + \varepsilon^2)(1 - \cos \omega T_0) - \varepsilon \tilde{\varepsilon} \omega T_0 \sin \omega T_0, \quad (10)$$

i.e.,

$$\frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2} = \frac{\gamma^2 + 1}{\gamma^2}(1 - \cos \omega T_0) - \frac{1}{\gamma^2} \omega T_0 \sin \omega T_0. \quad (11)$$

In a nonrelativistic case,  $\gamma^2 \rightarrow 1$ , and we have the same result as in [4]

$$\frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2} = \frac{K - K_0}{K \varepsilon^2} = 2(1 - \cos \omega T_0) - \omega T_0 \sin \omega T_0, \quad (12)$$

where  $K$  is the kinetic energy of the particle at time  $T$ ,  $K_0$  is the kinetic energy of the particle at entering the cavity.

Now let us consider the split-cavity oscillator (Figure 1). In the case of small signal analysis, the alternating electric field in the SCO's cavity can be written as [4]

$$\begin{aligned} E &= E_0 \sin(\omega t + \vartheta) \quad \text{for } 0 \leq z < g, \\ E &= -E_0 \sin(\omega t + \vartheta) \quad \text{for } g \leq x \leq 2g. \end{aligned} \quad (13)$$

Using (13), from the equation of motion similar to (2), we can obtain the following expression for particle's momentum and velocity in an SCO for the time interval  $t$  between the moments when the particle enters the SCO and when it crosses the foil placed at point  $z = g$ , i.e., when the inequality  $0 \leq t \leq T_g$  holds (here  $T_g$  is the moment of time when the particle crosses the foil):

$$\gamma v = F_{1SCO}(t, \vartheta) = \gamma_0 v_0 \{1 + \varepsilon([\cos(\omega t + \vartheta) - \cos \vartheta])\} \quad (14)$$

After the particle enters the second region of the SCO, the expression for

particle's momentum and velocity for the time interval  $T_g \leq t \leq T$  ( $T$  is the moment of time when the particle leaves the SCO structure, or the time needed for the particle to pass through the SCO) has the form:

$$\begin{aligned}\gamma v &= F_{2SCO}(t, \vartheta) \\ &= \gamma_0 v_0 \{1 + \varepsilon([\cos(\omega T_g + \vartheta) - \cos \vartheta] - [\cos(\omega t + \vartheta) - \cos(\omega T_g + \vartheta)])\},\end{aligned}\tag{15}$$

Equations (14) and (15) yield the equations defining the particle's trajectory  $z(t)$  that coincide in form with (6) with the function  $F(t, \vartheta)$  replaced by  $F_{SCO}(t, \vartheta)$ . As a result, the equation defining the time  $T$  needed for the particle to traverse the area occupied by the SCO has the form:

$$\int_0^T F_{SCO}(t', \vartheta) \frac{1}{\sqrt{1 + \frac{1}{c^2} F^2(t', \vartheta)}} dt' = 2g.\tag{16}$$

The function  $F_{SCO}$  can be written in the form:

$$\begin{aligned}F_{SCO} &= F_{1SCO} && \text{for the time } t \text{ in the interval } 0 \leq t < T_g, \\ F_{SCO} &= F_{2SCO} && \text{for the time } t \text{ in the interval } T_g \leq t \leq T.\end{aligned}$$

In the linear in  $\varepsilon$  approximation, (16) yields the follows expression for time  $T$

$$\begin{aligned}T &= \frac{2g}{v_0} - \tilde{\varepsilon} \frac{[\sin(\omega T_{g0} + \vartheta) - \sin \vartheta - \omega T_{g0} \cos \vartheta]}{\omega} \\ &\quad + \frac{\tilde{\varepsilon}}{\omega} [\sin(\omega T_0 + \vartheta) - \sin \vartheta (\omega T_{0g} + \vartheta) - \omega T_{g0} [2 \cos(\omega T_{g0} + \vartheta) - \cos \vartheta]],\end{aligned}\tag{17}$$

where

$$T_0 = \frac{2g}{v_0}, \quad T_{g0} = \frac{g}{v_0}.$$

As a result, using (15), we can obtain the expression  $p(T) = m\gamma(T)v(T)$  for particle momentum at time  $T$  and then, the following expression for the energy



change due to the interaction of a relativistic particle and the field  $E(t)$  in a SCO structure

$$\begin{aligned} \frac{E^2 - E_0^2}{p_0^2 c^2} &= \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{\gamma^2 v^2}{\gamma_0^2 v_0^2} - 1 \right) d\vartheta = \varepsilon \tilde{\varepsilon} (3 - 4 \cos \omega T_{g0} - 4 \omega T_{g0} \sin \omega T_{g0} \\ &+ \cos 2\omega T_{g0} + 2\omega T_{g0} \sin 2\omega T_{g0}) + \varepsilon^2 (3 - 4 \cos \omega T_{g0} + \cos 2\omega T_{g0}), \end{aligned} \quad (18)$$

i.e.,

$$\begin{aligned} \frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2} &= \frac{1 + \gamma^2}{\gamma^2} (3 - 4 \cos \omega T_{g0} + \cos 2\omega T_{g0}) \\ &- \frac{1}{\gamma^2} (4\omega T_{g0} \sin \omega T_{g0} - 2\omega T_{g0} \sin 2\omega T_{g0}). \end{aligned} \quad (19)$$

In a nonrelativistic case ( $\gamma^2 \rightarrow 1$ ), for the energy change we have the expression derived in [4]

$$\begin{aligned} \frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2} &= \frac{K - K_0}{K_0 \varepsilon^2} \\ &= [6 - 8 \cos \omega T_{g0} + 2 \cos 2\omega T_{g0} - 4\omega T_{g0} \sin \omega T_{g0} + 2\omega T_{g0} \sin 2\omega T_{g0}]. \end{aligned} \quad (20)$$

Expressions (11) and (19) enable the analysis of the energy change in TTO and SCO structures, depending on the parameters of the system for a relativistic case.

The curves in Fig. 3 show the characteristic beam instability region ( $\frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2} < 0$ ) when the transit time  $T_{g0}$  is about a quarter period [4]. However, in contrast to a nonrelativistic case, the depth of the instability region decreases as the particle energy is increased ( $\gamma$  is increased), and the instability drops sharply when the energy of electrons reaches the range from 250 to 400 keV. The instability region remains only for large transit times (right minimum).

But note must be taken of the fact that this result is valid until the beam

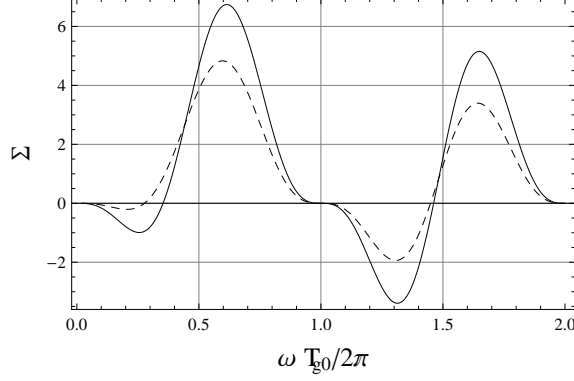


Figure 3. Energy change  $\left(\Sigma = \frac{E^2 - E_0^2}{p_0^2 c^2 \varepsilon^2 \omega T_{g0}}\right)$  versus the transit time  $T_{g0}$  for the initial energies  $E_0 = 50$  keV (solid) and  $E_0 = 200$  keV (dashed).

density is such that the effect of space charge on this type of instability can be neglected. According to [4], as the current of a nonrelativistic beam is increased, the space charge effect on instability enhances. Our analysis shows that in the case of relativistic beams, the effect of space charge on this type of instability also enhances with growing beam current. As a result, the beam self-modulation effect in SCO's occurs at energies greater than 250 keV too. This can be explained by the fact that when the beam density in SCO (TTO) structures rises, the beam instability grows, because the current in the beam approaches the range of values at which the vircator can be formed and oscillations in the vircator can be excited. As a result, the relativistic beam escaping the SCO appears to be modulated within a wide range of currents and space charge densities and their values can even be below the threshold for vircator formation.

It is well known that a charged particle passing through the medium-vacuum boundary emits transition radiation [8]. According to [9], the number of transition radiation quanta,  $N_{ph}$ , produced by a modulated beam can be written

in the form:

$$N_{ph} = N_e \left( N_{1ph} + c \frac{d^2 N_{1ph}}{d\omega d\Omega} \bigg|_{\vec{K}=\vec{\tau}} \frac{\pi}{2} \mu^2 \frac{\rho_0}{k_0^2} \right), \quad (21)$$

where  $N_e$  is the number of electrons that have passed through the medium-vacuum boundary,  $N_{1ph}$  is the number of quanta produced by one electron,  $k_0$  is the photon wave number, and  $\mu = \frac{\rho_1}{\rho_0}$ . It is assumed that the beam density is modulated according to the law :

$$\rho(z) = \rho_0 + \rho_1 \cos \tau z, \quad \tau = \frac{2\pi}{d},$$

where  $d$  is the spatial period of modulation. Here

$$\frac{d^2 N_{1ph}}{d\omega d\Omega} \bigg|_{\vec{K}=\vec{\tau}}$$

is the spectral-angular distribution of transition radiation at  $\vec{K} = \vec{\tau}$ , where  $\vec{\tau} = (0, 0, \tau)$ , and  $\vec{K} = (\vec{k}_\perp, \frac{\omega}{v})$ , where  $\vec{k}_\perp$  is the component of the photon wave vector that is orthogonal to the  $z$ -axis (parallel to the boundary between the medium and vacuum).

When the particle crosses the metal-vacuum boundary, the spectral-angular distribution of transition radiation has the form [8]:

$$\frac{d^2 N_{1ph}}{d\omega d\Omega} = \alpha \frac{v^2}{\pi^2 \omega c^2} \frac{1}{(1 - \frac{v^2}{c^2} \cos^2 \vartheta_{eph})^2} \quad (22)$$

where  $\alpha$  is the fine structure constant,  $\Omega$  is the solid angle,  $\vartheta_{eph}$  is the angle between the electron velocity and the photon escape direction. The radiation power  $P = \hbar \omega \dot{N}_{ph}$ , where  $\dot{N}_{ph}$  is the number of quanta emitted per unit time. The number of electrons escaping from the SCO per unit time is defined by

formula

$$\dot{N}_e = \frac{I}{e},$$

where  $I$  is the current of the outcoming electrons. Consequently,

$$\dot{N}_{ph} = \frac{I_e}{e} \left( N_{1ph} + c \frac{d^2 N_{ph}}{d\omega d\Omega} \bigg|_{\vec{K}=\vec{\tau}} \frac{\pi}{2} \mu^2 \frac{\rho_0}{K_0^2} \right), \quad (23)$$

According to (23), the power of coherent radiation depends significantly on the degree of modulation of the beam escaping from the SCO. Since the beam continues bunching for some time after it has escaped from the SCO, placing a foil or a wire mesh on the beam's path at some distance from the SCO might be beneficial for increasing the degree of beam modulation and hence the power of coherent transition radiation. The power of coherent transition radiation will increase after the beam has passed through such an obstacle.

The formulas derived here provide a means to evaluate the power of coherent radiation of a modulated beam. For example, the radiation power for the current  $I_e \approx 4 - 6$  kA and the energy of electrons from 400 to 500 keV appears to be of about several hundreds of megawatts at frequencies in the range of several gigahertz and the degree of beam modulation  $\mu \geq 0.5$ .

An important feature of the SCO structure is that there is no need for an applied magnetic field to guide a high-current beam. Let us recall in this regard that similar properties are demonstrated by structures in which one or several electron beams pass through periodically placed metallic threads, meshes (foils), or longitudinal/transverse posts forming a two- or three-dimensional diffraction grating (photonic crystal), being the resonator of a volume free electron laser (VFEL) [10,11]. Volume distributed feedback that is formed in such

structures not only provides the beam modulation and efficient generation, but also enables self-phase locked generation of independent beams passing through the structure of this type [12].

### **3 Conclusion**

Using the method of small signal analysis, we explored the application potential of relativistic electron beams in split-cavity oscillators (SCO). The change in the beam energy in SCO's as a function of the initial energy of a relativistic beam was considered. It has been shown that the small signal analysis method enables adequate evaluation of SCO parameters needed for effective modulation of a relativistic beam in a split cavity and for HPM generation using SCO's. It has been determined in what energy range the effect of self-modulation of the beam density in SCO structures is most pronounced. It has been demonstrated that for beam currents at which the space charge has little effect on the electron motion in the beam, the beam is effectively self-modulated in SCO's at beam energies less than  $\approx 300 \div 400$  keV. The self-modulation drops sharply in the range of energies from 250 to 400 keV, but as the beam current is increased, the effective beam self-modulation becomes appreciable in this range too, as well as even in a higher energy range.

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